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Behavior of the Turbulent Energy Equation at a Fixed Boundary

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I. Introduction

THE behavior of the turbulent energy equation at a fixed boundary has an important significance to the modeling of turbulence phenomena. Coantic[1] expressed the turbulent pressure and velocity components as a Taylor series with respect to distance from a wall. He concluded that the dissipation at the wall was approximately 0.095, when normalized by friction velocity (U_τ) and kinematic viscosity (ν). Townsend[2] corrected Laufer's[3] results to make dissipation balance gradient diffusion, with the other terms of the energy equation zero. He cited a dimensional argument to justify his correction. Hanjalić and Launder[4] used a similar power series to estimate dissipation near the wall.

The work reported in this Note is an extension of the previous work of Coantic[1] and Townsend,[2] and it is based on an internal axisymmetric incompressible turbulent flow. The object of this Note is to collect the scattered work of this type into a single source.

II. Analysis

A standard cylindrical coordinate system is used with variation of mean quantities being restricted to the radial direction. Upper case letters represent mean quantities, with lower case representing turbulent quantities; the symbols u, v, w represent the axial (x), radial (r), and transverse (ϕ) velocity components, respectively.

A power series expansion, with respect to distance from the wall ($y=R-r$), is used to describe the turbulence field. Subject to the continuity equation and the no-slip boundary conditions, the following expansions are used:

$$u=a_1(x,\phi,t)y+\ldots \tag{1}$$

$$v=b_2(x,\phi,t)y^2+\ldots \tag{2}$$

$$w=c_1(x,\phi,t)y+\ldots \tag{3}$$

$$p=d_0(x,\phi,t)+d_1(x,\phi,t)Y+\ldots \tag{4}$$

Eckelmann[5] reported fluctuations in the molecular shear stress, for directions parallel to the wall, at the wall. Hence u and w must contain a linear term.

The turbulent energy equation, in cylindrical coordinates, is

$$\begin{aligned}
&\overbrace{\frac{\nu}{U_\tau^4}\left[\overline{u^2}\frac{\partial U}{\partial x}+\overline{uv}\left(\frac{\partial U}{\partial x}+\frac{\partial U}{\partial r}\right)+\overline{v^2}\frac{\partial V}{\partial r}+V\frac{\overline{w^2}}{r}\right]}^{\text{Production}}\\
&+\overbrace{\frac{\nu}{U_\tau^4}\left[\frac{\partial}{\partial x}(U\overline{q^2})+\frac{1}{r}\frac{\partial}{\partial r}(rV\overline{q^2})\right]}^{\text{Advection}}\\
&+\overbrace{\frac{\nu}{U_\tau^4}\left[\frac{\partial}{\partial x}\overline{u(q^2+p)}+\frac{1}{r}\frac{\partial}{\partial r}r\overline{v(q^2+p)}\right]}^{\text{Diffusion}}\\
&-\overbrace{\frac{\nu^2}{U_\tau^4}\left[\frac{\partial^2}{\partial x^2}(\overline{q^2})+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\overline{q^2}\right)\right]}^{\text{Gradient Diffusion}}\\
&+\overbrace{\frac{\nu^2}{U_\tau^4}\left[\left(\overline{\frac{\partial u}{\partial x}}\right)^2+\left(\overline{\frac{1}{r}\frac{\partial u}{\partial\phi}}\right)^2+\left(\overline{\frac{\partial u}{\partial r}}\right)^2\right.}^{\text{Dissipation}}\\
&+\left(\overline{\frac{\partial v}{\partial x}}\right)^2+\left(\overline{\frac{\partial v}{\partial r}}\right)^2+\left(\overline{\frac{1}{r}\frac{\partial v}{\partial\phi}}\right)^2\\
&+\left(\overline{\frac{\partial w}{\partial x}}\right)^2+\left(\overline{\frac{\partial w}{\partial r}}\right)^2+\left(\overline{\frac{1}{r}\frac{\partial w}{\partial\phi}}\right)^2\\
&\left.-\frac{4}{r}\left(\overline{\frac{w}{r}\frac{\partial v}{\partial\phi}}+\frac{\overline{v^2+w^2}}{r}\right)\right]=0
\end{aligned}\tag{5}$$

where $q^2=(u^2+y^2+w^2)/2$ and the over bar indicates a long time average.

When the power series is substituted into the energy equation and evaluated at the wall ($y=0$) the following results.

Production

$$\Big|_{y=0}=0 \tag{6}$$

Advection

$$\Big|_{y=0}=0 \tag{7}$$

Diffusion

$$\Big|_{y=0}=0 \tag{8}$$

Gradient Diffusion

$$\Big|_{y=0}=\frac{\nu^2}{U_\tau^4}\left(\overline{a_1^2(x,\phi,t)}+\overline{c_1^2(x,\phi,t)}\right) \tag{9}$$

Dissipation

$$\Big|_{y=0}=\frac{\nu^2}{U_\tau^4}\left(\overline{a_1^2(x,\phi,t)}+\overline{c_1^2(x,\phi,t)}\right) \tag{10}$$

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Table 1 Limiting values of the constants

Author	Kind of investigation	Reynolds number	DISM[a] $\nu^2/U_\tau^4\overline{a_1^2}$	R[b] $\nu^2/U_\tau^4\overline{c_1^2}$	GDIF[c] $\nu^2/U_\tau^4(\overline{a_1^2}+\overline{c_1^2})$	$\sqrt{\overline{a_1^2}/\overline{c_1^2}}$
Fortuna and Hanratty[6]	Pipe flow, aqueous solution electro-chemical method	1,400-1,700	0.09		0.0981[d]	3.3[d]
Sirkar and Hanratty[7]	As above	1,400-1,700		0.0081		
Py[8]	Channel flow, aqueous solution, electro-chemical method	5,000-23,000	0.09	0.0132	0.1032[d]	2.6[d]
Laufer[3]	Pipe flow, air hot-wire	50,000-500,000	0.0961	0.00052	0.88	13.6[d]
Sreenivasan and Antonia[9]	Channel flow, air hot-film	15,000-25,400	0.0625			
Klages[10]	Pipe flow, oil hot film	17,600-17,900	0.0420			
Eckelmann[5]	Channel flow, oil hot-film	5,600-8,200	0.0576			
Kreplin[11]	As above	4,800-7,100	0.0625			
Kreplin and Eckelmann[12]	As above	7,700	0.0625	0.0042	0.0667[d]	3.9[d]
Schildknecht et al[13]	Pipe flow, air hot wire	27,000-79,000	0.066	0.022[d]	0.088	1.7[d]
Mean			0.0699	0.010	0.089	

[a]Measureable dissipation. [b]Remainder term. [c]Gradient diffusion. [d]Calculated.

III. Determination of the Constants

The values of the constants $\overline{a_1^2}$ and $\overline{c_1^2}$, given in Table 1, have been obtained from various sources. Laufer[3] gives values of both gradient diffusion [Eq. (9)] and measureable dissipation $(\overline{a_1^2})$, but his dissipation measurement is incorrect. Using the slopes of the u' and w' (rms) distribution it is possible to estimate the constants. This approach has been used by Fortuna and Hanratty,[6] Sirkar and Hanratty,[7] Py,[8] Sreenivasan and Antonia,[9] Klages,[10], Kreplin,[11] and Kreplin and Eckelmann.[12] Schildknecht et al.[13] carried out an extensive measurement of the turbulent energy balance and obtained values of measureable dissipation and gradient diffusion near a wall.

Generally, the distribution of w' shows a sharp curvature at a wall making it very difficult to estimate $\overline{c_1^2}$. Although this is not true for the u' distribution, any measurement near a wall is difficult to interpret.

IV. Conclusions

From the preceding discussion it is possible to conclude that gradient diffusion balances dissipation at the wall. All other terms in the turbulent energy equation are zero at the wall.

Experimentally we recognize that gradient diffusion approaches ~ 0.09, and measureable dissipation equals ~ 0.07 at the wall. These values apparently can be measured quite accurately. The value of the remainder term should equal ~ 0.02 at the wall. Evidently, these values are subject to change with improvement of experimental technique; but, it is unlikely that they will change dramatically.

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